Meson Spectroscopy without Tetraquarks

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Abstract

Data on $e^+e^- \to \pi^+\pi^-\Upsilon(1S, 2S, 3S)$ show a large increase in branching fractions near $\Upsilon(10860)$. A suggestion of Ali et al. [1] is to interpret this as evidence for a tetraquark, $Y_b(10890) \equiv (bn)(\bar{b}\bar{n})$. However, it may also be interpreted in terms of $\Upsilon(10860) \to \bar{B}B^*$, $\bar{B}^*B^*$ and $\bar{B}_s B_s^*$ above the open-b threshold, followed by de-excitation processes such as $\bar{B}B^* \to \Upsilon(1S, 2S, 3S)$. In the charm sector, a hypothesis open to experimental test is that X, Y and Z peaks in the mass range 3872 to 3945 MeV may all be due to regular $^3P_1$ and $^3P_2$ $\bar{c}c$ states (and perhaps $^3P_0$ and $^1P_1$) mixed with meson-meson.

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1 Introduction

There is much speculation about the existence of tetraquarks. Most papers unfortunately do not distinguish between tetraquarks and simple meson-meson. It is prudent, but less exciting, to examine conventional interpretations in terms of non-exotic $\bar{n}n$, $\bar{s}s$, $\bar{c}c$ or $\bar{b}b$ states with admixtures of meson-meson.

Nuclei may be viewed as six quark combinations. However, nucleon-nucleon phase shifts may be fitted in terms of meson exchanges. The essential reason nuclear matter does not collapse is short-range repulsion due to the Pauli principle. Returning to meson spectroscopy, the conventional $\bar{q}q$ states appear as octets and singlets (or linear combinations). Jaffe has suggested that two coloured quarks can form a colour $\bar{3}_c$ which is anti-symmetric, or 6_c which is symmetric [2]. Then 0^+ mesons can be formed as $(\bar{3}3)$ combinations of flavour and colour. This neatly evades repulsion between qq and also accounts for the high masses of $a_0(980)$ and $f_0(980)$. However, Jaffe's scheme without meson-meson disagrees with observed branching ratios [3]. The ratio $g^2(f_0(980) \to KK)/g^2(a_0(980) \to KK)$ (where g are coupling constants) requires that $f_0(980)$ has a dominant $K\bar{K}$ component; also the predicted ratio $g^2(\kappa \to K\pi)/g^2(\sigma \to \pi\pi)$ is much too small. What may defeat Jaffe's proposal is chiral symmetry breaking. The pion and kaon are abnormally light, with the result that meson-meson configurations $(\bar{q}q)(\bar{q}q)$ dominate over $(qq)(\bar{q}\bar{q})$ in σ, κ, $a_0(980)$ and $f_0(980)$. Present measurements of branching ratios are not good enough to rule out some small admixture of $(qq)(\bar{q}\bar{q})$; further improvements in the branching ratios of σ and $f_0(980)$ to KK, $\eta\eta$ and $\pi\pi$ above 1 GeV would help greatly.

Meson exchanges do make good predictions for $\pi\pi$ [4] and $K\pi$ [5] phase shifts up to masses where $q\bar{q}$ resonances appear. They also predict correctly low energy $I = 2$ and $I = 3/2$ phase shifts. There is no evidence for the **27** representation predicted for $(6, 6)$ combinations of flavour and colour. In a valuable review, Richard [6] discusses the issues in terms of the flux-tubes within four-quark configurations.

There is a further feature relevant to meson-meson configurations. The train of argument is as follows. Many mesons are observed at or very close to thresholds of opening channels. Well

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known examples are $f_0(980)$ and $a_0(980)$ at the KK threshold, $f_2(1565)$ ar the $\omega\omega$ threshold and $X(3872)$ at the $\bar{D}_0\bar{D}^*$ threshold within ~ 0.3 MeV. The mechanism for this synchronisation is rather fundamental [7]. The conventional form for the denominator of a resonance is

$$D(s) = M^2 - s - i\sum_i G_i^2 \rho_i(s), \tag{1}$$

where s is Lorentz invariant mass squared, $G_i = g_i F_i(s)$, g_i are coupling constants of open channels and F_i are form factors. However, the correct form for $D(s)$ is $M^2 - s - \operatorname{Re}\Pi(s) - i\operatorname{Im}\Pi(s)$ where $\operatorname{Im}\Pi(s) = G_i^2(s)\rho_i(s)$ and

$$\operatorname{Re}\Pi(s) = \frac{1}{\pi}\mathrm{P}\int_{s_{\mathrm{thr}}}^{\infty} \mathrm{ds}' \sum_i \frac{G_i^2(s')\rho_i(s')}{s' - s}. \tag{2}$$

Here s_{thr} is the value of s at the opening of the threshold and P denotes the principal value integral; [$\operatorname{Im}\Pi(s)$ is the pole term from this expression.] The origin of Eq. (2) is that amplitudes are analytic functions of s, so that any change in the imaginary part of the amplitude must be accompanied by a change in the real part, or vice versa.

Fig. 1 sketches the behaviour of $\operatorname{Im}\Pi(s)$ and $\operatorname{Re}\Pi(s)$ at an S-wave threshold, KK in this example. The value of $\operatorname{Re}\Pi(s)$ is large and peaks exactly at the S-wave threshold. It acts as an attractor and can explain why $f_0(980)$ and $a_0(980)$ lie very close to the KK threshold. The present limitation in calculating $\operatorname{Re}\Pi(s)$ accurately is that form factors $F_i(s)$ are poorly known. Incidentally, Eq. (2) is equivalent to solving Schrödinger type equations, a procedure adopted by many authors. These solutions are explicitly analytic. It is also included in the model of van Beveren and Rupp, because their amplitudes are constructed algebraically to be analytic [8].
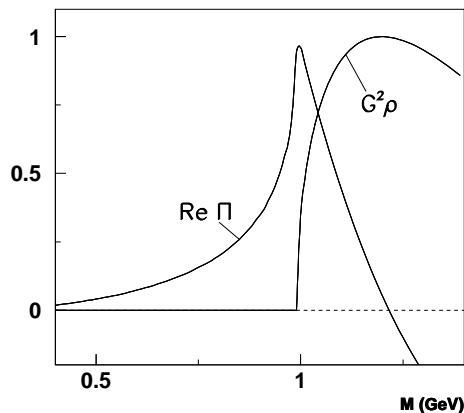

Figure 1: $\operatorname{Re}\Pi_{KK}(s)$ and $g_{KK}^2\rho_{KK}(s)$ for $f_0(980)$, normalised to 1 at the peak of $g_{KK}^2\rho_{KK}$.

The Hamiltonian for a $\bar{q}q$ state decaying to meson-meson obeys

$$H\Psi = \begin{pmatrix} H_{11} & V \\ V & H_{22} \end{pmatrix}\Psi; \tag{3}$$

H_{11} describes short-range $\bar{q}q$ components and H_{22} refers to ingoing and outgoing mesonic channels and must include t- and u-channel meson exchanges; V accounts for the coupling between them due to s-channel decays. The eigenfunction Ψ becomes a linear combination of $\bar{q}q$ and

meson-meson. The latter is not an 'optional extra'; if a resonance decays to meson-meson, that component is a necessary part of the wave function, and indeed plays a vital role. This is the basis of a large number of papers by Oset and collaborators, enumerating the attractive meson-meson contributions to a large number of mesons, e.g. [9]; their calculations include empirical short-range terms which may well simulate $\bar{q}q$ contributions.

The form of Eq. (3) is strictly analogous to the formation of covalent bonds in chemistry [10]. According to the variational principle, the eigenstate minimises the eigenvalue. The $\bar{q}q$ component is of short range. Mixing with meson-meson components at longer range lowers momentum components in the wave function and hence the energy eigenvalue.

The J/Ψ, $\Psi'(1S)$, $\Upsilon(1S)$ and $\Upsilon(2S)$ are very narrow and it is a good approximation to view them as pure $\bar{c}c$ states. Some commentators then argue that other mesons cannot be $\bar{c}c$ or $\bar{b}b$ because they have unexpected meson-meson components. That is wrong. If a resonance is close to an opening threshold with the same quantum numbers, it must contain virtual components of that channel. Taking $c\bar{c}n\bar{n}$ as an example, there are attractive long range mesonic components in the configurations $(c\bar{n})(\bar{c}n)$ and $(c\bar{c})(n\bar{n})$; examples are $D\bar{D}^*$ and $\omega J/\Psi$, which appear prominently in observed decays of X,Y,Z in the mass range 3872 to 3945 MeV. The observed meson-meson decays are in fact a signature of mesonic components in the wave function. Weinstein and Isgur modelled the $f_0(980)$ and $a_0(980)$ in terms of meson exchanges [11], as did the Julich group of Janssen et al. [12].

These general remarks set the scene for further details. Section 2 discusses $\Upsilon(10860)$ in this light. It might be the first clear tetraquark. However, it may also be understood at least qualitatively as a $\bar{b}b$ state coupled strongly to $\bar{B}B^*$, $\bar{B}^*B^*$ and $\bar{B}_s^*B_s^*$ channels which open in the mass range 10559 to 10826 MeV and can de-excite to $\pi\pi\Upsilon(nS)$, $n < 5$.

Section 3 discusses states popularly known as X, Y, Z. The Particle Data Group [13] labels them as $\chi_{c2}(2P)$, $X(3940)$ and $X(3945)$; the last of these is somewhat confusing in view of the fact that its average mass is now 3915 MeV. Here $Z(3930)$ will be used as a shorthand for $\chi_{c2}(2P)$. All these states lie close to the $\bar{D}D^*$ threshold. This leads to my suggestion that all of these states are $n = 2$ $\bar{c}c$ P-states mixed with meson-meson.

Spin-parity determinations of these states are presently lacking. This appears to be due to the fact that only decays have been analysed; that leads to ambiguities. It is essential to analyse the full process of production and decay, as in the analysis of Dalitz plots. This leads to orthogonal matrix elements between all J^P. The ideal process for analysis is $B \to K(\omega J/\Psi)$; here B and K both have spin 0, making the formulae rather simple. Formulae for this process were published in 2005 [14], but experimental groups have not taken advantage of spin information from decays of ω and J/Ψ; that is essential because it determines spin alignments. With the full formulae, my simulation (which assumes uniform geometric acceptance) suggests that ~ 50 events are sufficient to determine J^P if only a single amplitude is present or dominant. If there are more amplitudes, it gets harder, but it would still be valuable to know the outcome.

Section 4 reviews the Partial Wave Analysis using tensor notation in its simplest form. The formulae can be used in fully relativistic form. However, for the mass range 3872 to 3945 MeV, relativistic corrections are small and barely significant with present statistics. Two appendices discuss the tensor notation, also the simplest choice of axes and questions of how to do Lorentz transformations and rotations of axes, if needed. Section 6 summarises conclusions briefly.
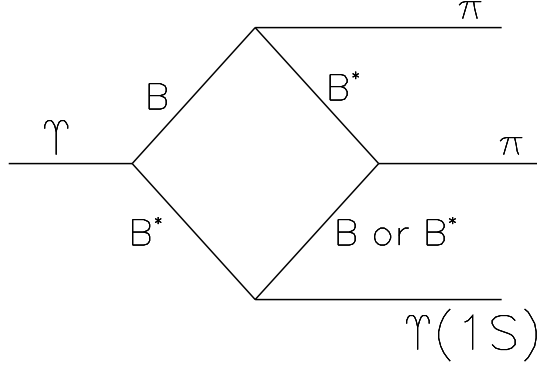
Figure 2: One diagram for $\Upsilon(10860) \to \pi\pi\Upsilon(1S)$.

2 The $\Upsilon(10860)$

The Belle Collaboration reports branching fractions for $\Upsilon(10860) \to \pi^+\pi^-\Upsilon(1S)$, $\pi^+\pi^-\Upsilon(2S)$ and $\pi^+\pi^-\Upsilon(3S)$ a factor ~ 50 larger than for $\Upsilon(4S)$ [15] [16]. Ali, Hambrook, Ahmed and Aslam interpret these branching fractions in terms of a tetraquark $Y_b(10890) \equiv (bn)(\bar{b}\bar{n})$ [1]. More recently, Ali, Hambrock and Mishima [17] predict for this configuration an intensity ratio

$$\sigma[\Upsilon(1S)K^+K^-]/\sigma[\Upsilon(1S)K^0\bar{K}^0] = 1/4. \tag{4}$$

For a $\bar{b}b$ or $\bar{B}B^*$ composition, this ratio should be 1, allowing a clear experimental test.

There is a natural explanation of the sizable branching fraction for $\Upsilon(10860) \to \pi\pi\Upsilon(1S, 2S, 3S)$ in terms of diagrams like that in Fig. 2. The intermediate states are off-shell, allowing π production, even though B^* and B_s^* are below the kinematic threshold for pionic decays. The final step allows de-excitation of $B^*\bar{B}$ to lower lying $\pi\pi\Upsilon$ channels with larger phase space. Many similar diagrams exist with intermediate B, B^*, B_s and B_s^*. Via mixing with meson-meson configurations, the wave function of $\Upsilon(10860)$ becomes

$$|\Upsilon(10860) >= \alpha|b\bar{b}> + \sum_{ij} \beta_{ij}|\bar{B}_iB_j> + \sum_{n=1,3} \gamma_n|\pi\pi\Upsilon(nS) > . \tag{5}$$

However, it is not possible to calculate these branching fractions accurately for lack of experimental information. The decay width of $\Upsilon(4S)$ to $\pi\pi\Upsilon(1S)$ is very weak, probably because it decays almost entirely to $B\bar{B}$. Transitions between B and $B\pi$ are forbidden by angular momentum conservation. Channels involving real B^* and B_s^* are closed, so only virtual intermediate states contribute. However, once intermediate $\bar{B}B^*$, $\bar{B}^*B^*$, $\bar{B}_sB_s$ and $\bar{B}_sB_s^*$ channels are open, there are 20 alternative diagrams. There may be complicated interferences amongst them. The conclusion is that the observed $\pi^+\pi^-\Upsilon(1S)$ decays of $\Upsilon(10860)$ are not dramatically out of line with expectation, bearing in mind the increase in phase space. What would be decisive is an accurate determination of the branching ratio of $\Upsilon(10860)$ to K^+K^- and $K^0\bar{K}^0$.

A technical detail concerns the fit to the $\pi^+\pi^-$ angular distribution. The $\pi^+\pi^-$ amplitude contains a broad pole at $(440 - 470)$ MeV. In some sets of data it appears as a peak, e.g. in $J/\Psi \to \omega\pi^+\pi^-$ [18]. In others it appears as in elastic scattering. In principle, it therefore needs to be fitted by an amplitude $[\Lambda_1 + \Lambda_2(s - s_A)]/D(s)$, where s_A is the Adler zero, 0.0097 GeV^2

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and Λ are coupling constants. An algebraic parametrisation of the denominator $D(s)$ is given in Ref. [19]. It is not correct to fit the data with the E791 parametrisation, a Breit-Wigner amplitude of constant width. That has a phase variation in complete disagreement with $\pi\pi$ elastic scattering and violates Watson's theorem [20]. Furthermore, it contains an unphysical pole just below the $\pi\pi$ threshold.

3   X,Y,Z

It is well known to theorists that the opening of strong thresholds can move resonances by large amounts. As one example, Barnes [21] has studied charmonium loop diagrams and concluded that mass shifts are potentially > 100 MeV; he finds that they tend to be similar for all states in a multiplet with similar radial excitation number n but different orbital angular momentum L, e.g. the $n = 2$ $^3P_{0,1,2}$ and $^1P_1$ states.

The $Z(3930)$ is identified clearly in Belle data for $e^+e^- \to e^+e^- + \bar{D}D$ by a strong D-wave component in $\bar{D}D$ with mass $M = 3929\pm5\pm2$ MeV, $\Gamma = 29\pm10\pm2$ MeV. [22]. It is now adopted by the PDG as the $\chi_{c2}(2P)$ radial excitation of χ_{c2} [13]. There are two observations still labelled $X(3945)$ by the PDG. One is by Babar in $B \to K(\omega J/\Psi)$ with $M = 3914.6^{+3.8}_{-3.4} \pm 2.0$ MeV, $\Gamma = 34^{+12}_{-8}\pm5$ MeV [23]. The second, with less events (49 ± 15), is by Belle in $e^+e^- \to e^+e^-(\omega J/\Psi)$ with $M = 3916 \pm 3 \pm 2$ MeV, $\Gamma = 17 \pm 10 \pm 3$ MeV [24]. The mass difference from $Z(3930)$ is $14\pm6\pm4$ MeV, so it is quite possible this is the $\chi_{c2}(2P)$ decaying to $\omega J/\Psi$. Another possibility is the $\chi_{c0}(2P)$.

There is considerable scatter in masses predicted for these states. A representative collection of recent predictions is shown in Table 1. Mass differences between $^3P_2$ and $^3P_0$ radial excitations are large, but can be affected by thresholds and form factors. If the $n = 2$ $^3P_2$ predictions can be trusted, at least approximately, this state has been pulled down in mass by a large amount. The strong signal for $\omega J/\Psi$ at 3915 MeV is a hint of mixing between $c\bar{c}$ and the $\omega J/\Psi$ channel and attraction to this threshold and/or the $D\bar{D}^*$ threshold.

State	BGS [25]	EFG [26]	[27]
$^3P_2$	3972	3972	3941
$^3P_1$	3925	3929	3900
$^3P_0$	3852	3854	3839
$^1P_1$	3934	3945	3909
$^1D_2$	3799	3811	3799

Table 1: Recent predictions of masses of $n = 2$ ^3P states and $n = 1$ $^1D_2$ in MeV.

The $X(3945)$ claimed by Belle [28] at $3943 \pm 11 \pm 13$ MeV with $\Gamma = 87 \pm 22 \pm 26$ MeV does not agree well with masses near 3915 MeV. However, they are in the same decay channel and with a small stretch of the errors may both be consistent with $Z(3930)$.

A puzzle is where the $^3P_0$ state is. Its obvious decay mode is to $\bar{D}D$. Its width is strongly dependent on the form factor for the $\bar{D}D$ channel [29],[30]. An early paper of Babar comments that separation of $D\bar{D}$ events is obscured by mis-identified $\bar{D}D^*$ events, so it could have escaped

detection just below that threshold [31]. The other obvious possibility is that it could be the 3915 MeV peak in $\omega J/\Psi$, but then the inference is that it couples strongly to that threshold.

The $X(3940)$ is observed by Belle in $e^+e^- \to J/\Psi +$ an inclusive collection of $\bar{D}D$ and $\bar{D}D^*$ events. From 52 identified $\bar{D}D^*$ events, Belle find a mass of $M = 3943^{+7}_{-6} \pm 6$ MeV [32]. There are two obvious possibilities for its spin-parity. Firstly, it could be the missing $^1P_1$ state. Secondly, it could be $\chi_{c2}(2P)$ decaying to $\bar{D}D^*$ with orbital angular momentum $\ell = 2$, though there is presently no claim to observe it in $\bar{D}D$. The rising phase space for $\ell = 2$ would naturally move the peak up in mass. To illustrate this, suppose $Z(3930)$ is described by a Breit-Wigner resonance of constant width with Belle parameters. A simple calculation using $\bar{D}D^*$ phase space and an $\ell = 2$ centrifugal barrier for decay with a radius of interaction of 0.73 fm [33] gives a peak at 3939 MeV with width 43 MeV; this is close to Belle parameters, but perhaps fortuitous in view of experimental errors for both $Z(3930)$ and $X(3940)$. Also strong coupling to the $\bar{D}D^*$ channel would move the peak by an amount which cannot be calculated without knowledge of the relative branching ratios to $\bar{D}D$ and $\bar{D}D^*$. A puzzle is that Belle do not observe any significant $B \to K(\bar{D}D^*)$ signal near 3940 MeV [34].

The spin-parity analysis of $e^+e^- \to J/\Psi + 2^+$ involves spins $1^- \to 1^- + 2^+ + L$; since momenta in the final state are large, L can run up to 4. This makes a complete partial wave analysis impossible with present data and $J^P = 0^+$ can be confused in this reaction with the helicity 0 component of the $^3P_2$ state.

4 Spin-Parity Analysis

Spin-parity determinations of these states are needed if any progress is to be made in understanding them. The cleanest route to J^P determination lies in $B \to K + X$, $X \to \omega J/\Psi$ (or $\rho J/\Psi$) or $\bar{D}D^*$. This is because both B and K are spinless, with the result that the kaon is produced with known orbital angular momentum equal to the spin of X; this simplifies the partial wave analysis greatly. However, that analysis *must* use formulae which describe both production and decay of X and must use spin information from both ω and J/Ψ or from the D^*. The spin of the ω is normal to its decay plane in the rest frame of $\pi^+\pi^-\pi^0$. The J/ψ is detected via decays to lepton-lepton, and its polarisation vector ϵ is normal to the lepton-lepton axis in its rest frame. These are key pieces of information. With them, matrix elements for the full amplitude contain correlations between the spectator kaon and decays of ω and J/Ψ which are highly distinctive; without these correlations, most of the relevant information is discarded. Formulae for these matrix elements were published in 2005 [14] and will be reviewed here in outline, together with comments on the simplest choice of reference frames.

If formulae are written in terms of vectors and tensors, the vital features are transparent. The formulae can be written in any reference frame, but in practice they simplify greatly in the rest frame of X,Y,Z for reasons explained algebraically in Appendix 1. A general point is that all matrix elements are orthogonal; that is a valuable check on computer programmes.

The decay of the resonance X is described by combined spins s of ω and J/Ψ (or $D + D^*$). The $Z(3930)$ is likely to decay dominantly with $\ell = 0$, though $l = 2$ is allowed in principle. However, the centrifugal barrier factor for $\ell = 2$ is quite strong. Table 2 exhibits Blatt-Weisskopf centrifugal barrier factors for the amplitude using a generous radius of interaction 0.73 fm, including the convolution of both particles in the final state; (average masses are used for D

Mass (MeV)	$\bar{D}D^*$ barrier	$\omega J/\Psi$ barrier
3900	0.1905	0.1105
3915	0.2880	0.1835
3930	0.3679	0.2498
3945	0.4361	0.3096
3960	0.4937	0.3635

Table 2: $L = 2$ centrifugal barriers for $\bar{D}D^*$ and $\omega J/\Psi$.

and D^*). For a smaller radius, the barrier factors are even smaller. The $\ell = 4$ barrier makes that amplitude negligible. The primary objective of the analysis should be to determine the dominant amplitudes, hence should start with $J^P = 0^+$ and 2^+, both with $\ell = 0$. Further $\ell = 2$ amplitudes may then be introduced one by one with penalty functions which constrain their magnitudes unless the data really demand them. This may be done, for example, by adjusting the penalty function to contribute χ^2 or log likelihood in the range 1–4.
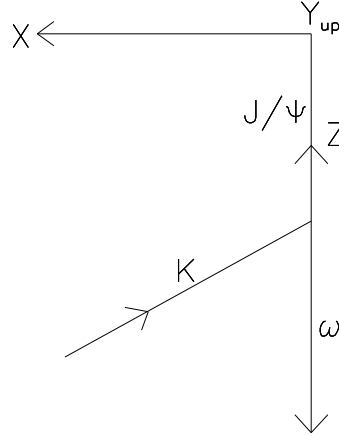

Figure 3: Axes describing $B \to K\omega J/\Psi$.

A simple choice of axes is shown in Fig. 3 with the Z-axis along the direction of the J/Ψ and Y along the normal to the plane of K, ω and J/Ψ. The formula for $\omega \to \pi^+\pi^-\pi^0$ is a 4-vector W given in Lorentz invariant form in Section 4.2 of [14]. This formula is easily evaluated in any reference frame, such as that of Fig. 3. It includes naturally the well-known enhancement near the edge of the Dalitz plot for its decay to $\pi^+\pi^-\pi^0$.

The J/Ψ decay is simplest in its rest frame. The effect of the Lorentz transformation to the rest frame of X,Y,Z is small because the J/Ψ is heavy and therefore non-relativistic in that frame. The gymnastics required for the Lorentz transformation of its polarisation vector ϵ are given in Appendix 2.

For $J^P = 1^+$, the kaon in the production process carries $L = 1$. In the resonance rest frame, this angular momentum is described by the 3-momentum $\vec{K}$ of the kaon. In the decay $X(3872) \to \rho J/\Psi$, the spin 1 of the ρ is represented by the vector $P = k(\pi_1) - k(\pi_2)$, where k are momenta in any reference frame. In the non-relativistic limit the full matrix element reduces

to $M = \vec{\epsilon}.\vec{P} \wedge \vec{K}$. There is a simple trick which can be used to do the spin average over $\vec{\epsilon}$. This is to write $\epsilon_x = 1$ and $\epsilon_y = i = \sqrt{-1}$; then intensities are obtained by taking the modulus squared of M. For $X(3872)$, $\ell = 2$ in the decay is eliminated by the tiny width of the state. The threefold dependence on the lepton axis, the spin of the ρ and the kaon direction is then unique and highly distinctive.

For $J^P = 0^+$, the matrix element is

$$M = \vec{W}.\vec{\epsilon} = W_1\epsilon_1 + W_2\epsilon_2 + W_3\epsilon_2 - W_4\epsilon_4, \tag{6}$$

where $\vec{W}$ and $\vec{\epsilon}$ are 4-vectors. Without the last term, which is very small, the matrix element is just the scalar product of 3-vectors $\vec{W}$ and $\vec{\epsilon}$. With this simplification, the intensity is $\sin^2 \alpha$, where α is the angle between $\vec{W}$ and the lepton axis. The simplicity of this formula may actually make the 0^+ state harder to identify than other amplitudes.

For $J^P = 2^+$, the kaon carries $L = 2$ and is described by a tensor

$$\tau_{\alpha\beta} = K_\alpha K_\beta - (1/3)\delta_{\alpha\beta}(K^2). \tag{7}$$

Here δ is the usual Kronecker delta function and $K^2 = K_1^2 + K_2^2 + K_3^2 - K_4^2$. For decays with $\ell = 0$, the combined spin of ω and J/Ψ is $s = 2$. The decay is then described by the tensor

$$T_{\alpha\beta} = \epsilon_\alpha W_\beta + \epsilon_\beta W_\alpha - (2/3)(\epsilon_\mu W^\mu) \tag{8}$$

in the resonance rest frame; $\epsilon_\mu W^\mu$ implies the summation over $\mu = 1$ to 4 and takes the same general form as K^2 above. The full matrix element is just the contraction $\tau_{\alpha\beta}T^{\alpha\beta}$. It is very distinctive because of the dependence on angles of all of K, $\vec{W}$ and $\vec{\epsilon}$.

A simulation of these J^P, assuming full geometrical acceptance, suggests that 50 events would give a separation at 90% confidence level between 0^+, 1^+ and 2^+ if only one of them contributes a significant amplitude. Babar and Belle between them have many more events than this, with the exception of $X(3940) \to \bar{D}D^*$. If more than one J^P contributes, the separation is obviously poorer, and an overlap between 0^+ and 2^+ could be a difficult case. Formulae for $\ell = 2$ in decays are given in Ref. [14].

A distinctive feature of $X(3872)$ is that its mass lies within 0.3 MeV of the $\bar{D}_0 D_0^*$ threshold. There is a simple explanation. It is attracted to this threshold by the dispersive effect due to the opening of the threshold. Lee et al. [35] model the $X(3872)$ including exchange of π and σ between $\bar{D}$ and D^*. They solve the Schrödinger equation, including the $\bar{D}D^*$ channel; this is equivalent to evaluating the dispersion integral of Eq. (2), since their solution is explicitly analytic. One should not be misled into thinking that the effect of $\bar{D}D^*$ is small because few events are seen in $X(3872) \to \bar{D}_0 D_0^*$; there are few events because of the narrow width of $X(3872)$. However, the attraction to the threshold does not involve the resonance denominator in Eq. (2); it arises from the integral over all phase space for $\bar{D}D^*$. A parallel example is the deuteron. Its binding energy is not governed by its tiny width; it comes from virtual meson exchanges over the whole left-hand cut.

Present data have failed to distinguish between $J^P = 1^+$ and 2^- for $X(3872)$. The key point is to check whether partial wave analysis is consistent with $J^P = 1^+$ or not. If it is, the fact that the mass coincides with the $\bar{D}D^*$ threshold is significant information. A state with $J^P = 2^-$ couples to $\bar{D}D^*$ with $l = 1$ and there is no obvious reason why it should lie close to this

threshold. In fact, predictions for the mass of $\eta_{c2}(1\,^1D_2)$ in Table 1 lie far below $X(3872)$; this is because spin-splitting for D states is smaller than for P-states and the mass of the $\psi(3770)$ (mostly $n = 1\,^3D_1$) is well known. For $J^P = 2^-$, formulae are more messy and are given in Ref. [14]. There are are in principle three amplitudes, but one requires $\ell = 3$, which is irrelevant for the narrow $X(3872)$. The others have $\ell = 1$ and combined spins either $s = 1$ or 2.

A full partial wave analysis of Belle data for $X(3940) \to \bar{D}D^*$ will in general be ambiguous with only ~ 50 events. However, this may be enough events to test the possibility that $J^P = 0^-$. The spin information is carried by the vector

$$Q = k_D - k_\pi - \frac{M_D^2 - m_\pi^2}{M_{D^*}^2}[k_D + k_\pi], \tag{9}$$

where k are momenta (in any frame). With $J^P = 0^-$, production requires $L = 1$ and decays to $D\bar{D}^*$ with $\ell = 1$, so only a single amplitude is present; it has the distinctive feature of going through zero at the centre of the angular distribution for decays. There is a missing $^1P_1$ $\bar{c}c$ state. From predicted masses in Table 1 and the fact that the mass of the $n = 1$ $^1P_1$ state is slightly about the $^3P_1$ state, it seems likely that the $^1P_1$ state will lie above $X(3872)$. However, it is not easy to find. It has charge conjugation $C = -1$, while $\omega J/\Psi$ has $C = +1$, so this decay is forbidden. It can appear in $\bar{D}D^*$. However, it appears in that channel in the combination $\bar{D}_+D^*_- + \bar{D}_0D^*_0$ (because of the minus sign associated with $\bar{D}_0$), while $^3P_1$ decays with the opposite relative sign. Without interferences with a state of established J^{PG}, $^1P_1$ and $^3P_1$ are not separated in any particular charge state.

5   Conclusions

It is necessary to be specific about what tetraquark configuration is meant in every discussion of them. Meson-meson combinations mixed with $q\bar{q}$ are to be expected via t- and u-channel exchanges and also from the s-channel decay.

The increase in branching fractions for $e^+e^- \to \pi^+\pi^-\Upsilon(1S, 2S, 3S)$ as the mass increases from $\Upsilon(4S)$ to $\Upsilon(10860)$ may be understood qualitatively via the opening of real decays to $\bar{B}B^*$, $\bar{B}^*B^*$ and $\bar{B}_sB^*_s$ channels in the mass range 10559 to 10826 MeV, followed by de-excitation to $\pi\pi\Upsilon$. Observed branching fractions are not seriously out of line with expectation. The composition $(bn)(\bar{b}\bar{n})$ may be tested by measuring the intensity ratio given by $\sigma[\Upsilon(1S)K^+K^-]/\sigma[\Upsilon(1S)K^0\bar{K}^0)$, which Ali, Hambrock and Mishima show to be $1/4$ for exotic tetraquarks; for a $\bar{B}B^*$ composition or $\bar{b}b$, this ratio should be 1.

Spin-parity analyses of X,Y,Z states are needed and can be done straightforwardly in several cases using formulae developed in Ref. [14]. It is suggested here that they can be explained naturally as the expected $n = 2$ radial excitations of $^3P_1$, $^3P_2$ and perhaps $^3P_0$. It is essential to make use of the full forms of matrix elements for production and decay for partial wave analysis.

6   Acknowledgement

7   Appendix 1

Chung gives an introduction to tensor formalism [36]. Foundation work was done by Zemach [37]. The convention used here is that the Lorentz metric $g_{\mu\nu}$ is such that four-vectors are written as $(-p_1, -p_2, -p_3, E = p_4)$, i.e. $g_{\mu\nu}$ has only diagonal elements (-1,-1,-1,1).

The decay of a scalar particle, e.g. $\sigma \to \pi\pi$, is described by its mass $(k_1 + k_2)_4 \equiv P_4$; here $k_{1,2}$ are 4-vectors of the decay pions. A spin 1 particle needs to be described by a vector constructed to be orthogonal to the scalar expression. This is done, for example in describing $\rho \to \pi\pi$, using

$$(k_{12}^{\perp})_\mu = g_{\mu\nu}^{\perp}(k_1 + k_2)_\nu, \tag{10}$$

where

$$g_{\mu\nu}^{\perp} = g_{\mu\nu} - (P_\mu P_\nu)/M_\rho^2. \tag{11}$$

Substituting (11) into (10),

$$(k_{12}^{\perp})_\mu = g_{\mu\nu}(k_1 - k_2)_\nu - \frac{[(k_1 - k_2)_\nu(k_1 + k_2)_\nu](k_1 + k_2)_\mu}{M_\rho^2}. \tag{12}$$

The orthogonality between vector and scalar is demonstrated from the dot product of $(k_{12}^{\perp})_\mu$ with $(k_1 + k_2)_\mu$:

$$g_{\mu\nu}(k_1 - k_2)_\nu(k_1 + k_2)_\mu - (M_1^2 - M_2^2)M_\rho^2/M_\rho^2 = 0. \tag{13}$$

In the rest frame of the decaying particle, $g_{\mu\nu}^{\perp}$ has diagonal elements [-1,-1,-1,1]-[0,0,0,1]= -[1,1,1,0]. This gives the important result that $k^{\perp}$ is a 3-vector in the rest frame of a resonance. Then $k^{\perp}$ describes the angular momentum carried by the particle. Tensor expressions for higher spins are built up as described by Zemach and Chung so that they are orthogonal to lower spins. They are symmetric and traceless, i.e. diagonal elements add to 0. We shall need only the tensor $T_{\mu\nu}$ for spin 2.

8   Appendix 2

Suppose the lepton pair from J/Ψ decay has components $(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ in axes in the J/Ψ rest frame parallel to X, Y, Z of Fig. 3. This vector may be expressed as

$$\begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{14}$$

The spin ϵ of the J/Ψ is orthogonal to this lepton axis. In axes x,y,z aligned along the lepton axis, ϵ is given by a vector $(\cos R, \sin R, 0)$ and eventually it is necessary to average over R. In X,Y,X, axes, ϵ is given by the converse relation to Eq. (14) with the result

$$\begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos R \\ \sin R \\ 0 \end{pmatrix} \tag{15}$$

$$= \begin{pmatrix} \cos\theta(\cos\phi\cos R + \sin\phi\sin R) \\ -\sin\phi\cos R + \cos\phi\sin R \\ \sin\theta(\cos\phi\cos R + \sin\phi\sin R) \end{pmatrix}. \tag{16}$$

Under the Lorentz transformation back to the centre of mass frame, this becomes

$$\begin{pmatrix} \cos\theta(\cos\phi\cos R + \sin\phi\sin R) \\ -\sin\phi\cos R + \cos\phi\sin R \\ \gamma\sin\theta(\cos\phi\cos R + \sin\phi\sin R) \\ \beta\gamma\sin\theta(\cos\phi\cos R + \sin\phi\sin R) \end{pmatrix}, \tag{17}$$

where γ and β are the usual expressions for the Lorentz transformation. This approach is simpler than that given in Ref. [14].

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